GRAVITATIONAL EXCITONS FROM EXTRA DIMENSIONS

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We study inhomogeneous multidimensional cosmological models with a higher dimensional space-time manifold $M = M_0 \times \prod_{i=1}^{n} M_i$ $(n \geq 1)$ under dimensional reduction to D_0-dimensional effective models and show that small inhomogeneous excitations of the scale factors of the internal spaces near minima of effective potentials should be observable as massive scalar particles (gravitational excitons) in the external space-time.

Gravitational excitons

We consider a multidimensional space-time manifold

$$M = M_0 \times M_1 \times \ldots \times M_n \tag{1}$$

with metric

$$g = g_{MN}(X) dX^M \otimes dX^M = g^{(0)} + \sum_{i=1}^{n} e^{2\beta^i(x)} g^{(i)}, \tag{2}$$

where x are some coordinates of the $D_0 = d_0 + 1$ - dimensional manifold M_0. Let manifolds M_i be d_i-dimensional Einstein spaces with metric $g^{(i)}$, i.e., $R\left[g^{(i)}\right] = \lambda^i d_i \equiv R_i$. Internal spaces $M_i \quad (i = 1, \ldots, n)$ may have nontrivial global topology, being compact (i.e. closed and bounded) for any sign of spatial topology.

With total dimension $D = 1 + \sum_{i=0}^{n} d_i$, κ^2 a D-dimensional gravitational constant, Λ - a D-dimensional bare cosmological constant and S_{YGH} the standard York-Gibbons-Hawking boundary term, we consider an action of the form

$$S = \frac{1}{2\kappa^2} \int\limits_M d^D X \sqrt{|g|} \left\{ R[g] - 2\Lambda \right\} + S_{add} + S_{YGH}. \tag{3}$$

The additional potential term

$$S_{add} = - \int\limits_M d^D X \sqrt{|g|} \rho(x) \tag{4}$$

is not specified and left in its general form, taking into account the Casimir effect, the Freund-Rubin monopole ansatz, or a perfect fluid. In all these cases ρ depends on the external coordinates through the scale factors $a_i(x) = e^{\beta^i(x)}$ $(i = 1, \ldots, n)$ of the internal spaces.

After dimensional reduction the action reads

$$S = \frac{1}{2\kappa_0^2} \int\limits_{M_0} d^{D_0} x \sqrt{|g^{(0)}|} \prod_{i=1}^{n} e^{d_i \beta^i} \left\{ R\left[g^{(0)}\right] - G_{ij} g^{(0)\mu\nu} \partial_\mu \beta^i \, \partial_\nu \beta^j + \right.$$

$$\left. + \sum_{i=1}^{n} R\left[g^{(i)}\right] e^{-2\beta^i} - 2\Lambda - 2\kappa^2 \rho \right\}, \tag{5}$$

where $\kappa_0^2 = \kappa^2/V_I$ and $V_I = \prod_{i=1}^n v_i = \prod_{i=1}^n \int\limits_{M_i} d^{d_i}y\sqrt{|g^{(i)}|}$ are the the D_0-dimensional gravitational constant and the internal space volume. $G_{ij} = d_i\delta_{ij} - d_i d_j$ $(i,j = 1,\ldots,n)$ defines the midisuperspace metric. Action (5) is written in the Brans-Dicke frame. Conformal transformation to the Einstein frame

$$g^{(0)}_{\mu\nu} = \Omega^2 \tilde{g}^{(0)}_{\mu\nu} = exp\left(-\frac{2}{D_0-2}\sum_{i=1}^n d_i\beta^i\right)\tilde{g}^{(0)}_{\mu\nu} \tag{6}$$

yields

$$S = \frac{1}{2\kappa_0^2}\int\limits_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\tilde{R}\left[\tilde{g}^{(0)}\right] - \bar{G}_{ij}\tilde{g}^{(0)\mu\nu}\partial_\mu\beta^i\,\partial_\nu\beta^j - 2U_{eff}\right\}, \tag{7}$$

where $\bar{G}_{ij} = d_i\delta_{ij} + \frac{1}{D_0-2}d_i d_j,\ (i,j = 1,\ldots,n)$, and the effective potential reads

$$U_{eff} = \left(\prod_{i=1}^n e^{d_i\beta^i}\right)^{-\frac{2}{D_0-2}}\left[-\frac{1}{2}\sum_{i=1}^n R_i e^{-2\beta^i} + \Lambda + \kappa^2\rho\right]. \tag{8}$$

We recall that ρ depends on the scale factors of the internal spaces: $\rho = \rho\left(\beta^1,\ldots,\beta^n\right)$. Thus, we are led to the action of a self-gravitating $\sigma-$model with flat target space and self-interaction described by the potential (8). It can be easily seen that the problem of the internal spaces stable compactification is reduced now to the search of models that provide minima of the effective potential (8).

The midisuperspace metric (target space metric) by a regular coordinate transformation $\varphi = Q\beta, \quad \beta = Q^{-1}\varphi$ can be turned into a pure Euclidean form

$$\bar{G}_{ij}d\beta^i\otimes d\beta^j = \sigma_{ij}d\varphi^i\otimes d\varphi^j = \sum_{i=1}^n d\varphi^i\otimes d\varphi^i\,. \tag{9}$$

An appropriate transformation $Q: \ \beta^i \mapsto \varphi^j = Q^j_i\beta^i$ is given e.g. by

$$\begin{aligned} \varphi^1 &= -A\textstyle\sum_{i=1}^n d_i\beta^i, \\ \varphi^i &= \left[d_{i-1}/\Sigma_{i-1}\Sigma_i\right]^{1/2}\textstyle\sum_{j=i}^n d_j(\beta^j - \beta^{i-1}), \quad i = 2,\ldots,n\,, \end{aligned} \tag{10}$$

where $\Sigma_i = \sum_{j=i}^n d_j$, $A = \pm\left[\frac{1}{D'}\frac{D-2}{D_0-2}\right]^{1/2}$ and $D' = \sum_{i=1}^n d_i$. So we can write action (7) as

$$S = \frac{1}{2\kappa_0^2}\int\limits_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\tilde{R}\left[\tilde{g}^{(0)}\right] - \sigma_{ik}\tilde{g}^{(0)\mu\nu}\partial_\mu\varphi^i\,\partial_\nu\varphi^k - 2U_{eff}\right\} \tag{11}$$

with effective potential

$$U_{eff} = e^{\frac{2}{A(D_0-2)}\varphi^1}\left(-\frac{1}{2}\sum_{i=1}^n R_i e^{-2(Q^{-1})^i{}_k\varphi^k} + \Lambda + \kappa^2\rho\right). \tag{12}$$

Let us suppose that this potential has minima which are localized at points $\vec{\varphi}_c, c = 1, ..., m : \left.\frac{\partial U_{eff}}{\partial \varphi^i}\right|_{\vec{\varphi}_c} = 0$. Then, for small field fluctuations $\xi^i \equiv \varphi^i - \varphi^i_{(c)}$ around the minima the potential (12) reads

$$U_{eff} = U_{eff}\left(\vec{\varphi}_c\right) + \frac{1}{2}\sum_{i,k=1}^{n} \bar{a}_{(c)ik}\xi^i\xi^k + O(\xi^i\xi^k\xi^l)\,, \tag{13}$$

where the Hessians $\bar{a}_{(c)ik} := \left.\frac{\partial^2 U_{eff}}{\partial \xi^i\, \partial \xi^k}\right|_{\vec{\varphi}_c}$ are assumed as not vanishing identically. The action functional (11) reduces now to a family of action functionals for fluctuation fields ξ^i

$$\begin{aligned} S = \quad & \tfrac{1}{2\kappa_0^2}\int\limits_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\tilde{R}\left[\tilde{g}^{(0)}\right] - 2U_{eff}\left(\vec{\varphi}_c\right) - \right. \\ & \left. -\sigma_{ik}\tilde{g}^{(0)\mu\nu}\partial_\mu\xi^i\,\partial_\nu\xi^k - \bar{a}_{(c)ik}\xi^i\xi^k\right\},\ c = 1, ..., m. \end{aligned} \tag{14}$$

It remains to diagonalize the Hessians $\bar{a}_{(c)ik}$ by appropriate $SO(n)-$rotations $S_c : \xi \mapsto \psi = S_c\xi, \quad S'_c = S_c^{-1}$

$$\bar{A}_c = S'_c M_c^2 S_c, \quad M_c^2 = \mathrm{diag}\ (m^2_{(c)1}, m^2_{(c)2}, \ldots, m^2_{(c)n}), \tag{15}$$

leaving the kinetic term $\sigma_{ik}\tilde{g}^{(0)\mu\nu}\partial_\mu\xi^i\,\partial_\nu\xi^k$ invariant

$$\sigma_{ik}\tilde{g}^{(0)\mu\nu}\partial_\mu\xi^i\,\partial_\nu\xi^k = \sigma_{ik}\tilde{g}^{(0)\mu\nu}\partial_\mu\psi^i\,\partial_\nu\psi^k, \tag{16}$$

and we arrive at action functionals for decoupled normal modes of linear $\sigma-$models in the background metric $\tilde{g}^{(0)}$ of the external space-time:

$$\begin{aligned} S \quad &= \quad \frac{1}{2\kappa_0^2}\int\limits_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\tilde{R}\left[\tilde{g}^{(0)}\right] - 2\Lambda_{(c)eff}\right\} + \\ &+ \quad \sum_{i=1}^{n}\frac{1}{2}\int\limits_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{-\tilde{g}^{(0)\mu\nu}\psi^i_{,\mu}\psi^i_{,\nu} - m^2_{(c)i}\psi^i\psi^i\right\},\ c = 1, \ldots, m, \end{aligned} \tag{17}$$

where $\Lambda_{(c)eff} \equiv U_{eff}\left(\vec{\varphi}_c\right)$ is the D_0-dimensional effective cosmological constant and the factor $\sqrt{V_I/\kappa^2}$ has been included into ψ for convenience: $\sqrt{V_I/\kappa^2}\psi \to \psi$.

Thus, conformal excitations of the metric of the internal spaces behave as massive scalar fields developing on the background of the external space-time. By analogy with excitons in solid state physics where they are excitations of the electronic subsystem of a crystal, the excitations of the internal spaces were called gravitational excitons [1].

References

1. U.Günther and A.Zhuk, *Phys. Rev.* D **56**, No.10 (1997), (in press), gr-qc/9706050.